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SENSITINITY ANALYSIS OF A LARGE CONSTRUCTING PROJECT

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ABSTRACT

The sensitivity analysis the validity ranges for the optimal objective function value and optimal basis. Although there is some benefit in predicting the effect of changes in data, it has been shown that these indicators do have their limits. Repeated solving of the model provides the best method of sensitivity analysis and the Aims modeling system has some powerful facilities to support this type of sensitivity analysis. The design and operation of large constructing project has become of concern to an ever increasing segment of the scientific and professional world. It is very difficult task to complete the selected project within the range of Budget resources. Large project involves a lot of activities and barrier to schedule and estimate. Sometimes we haven't enough money to complete a project work. In this work a new idea is developed, which is very effective to find out the maximum benefits by formulating a constructing project, applying linear programming and using simplex methods. The linear requirements and non-negativity conditions state that the variables cannot assume negative values. It is not possible to have negative resources.

Key words: shadow price, sensitivity, marginal cost, constraints, slicing, frying, and packing.

INTRODUCTION

In a linear program, slack variables may be introduced to transform an inequality constraint into an equality constraint. When the simplex method is used to solve a linear program, it calculates an optimal solution (i.e. optimal values for the decision and/or slack variables), an optimal objective function value, and partitions the variables into basic variables and non basic variables. Non basic variables are always at one of their bounds (upper or lower), while basic variables are between their bounds. The set of basic variables is usually referred to as the optimal basis and the corresponding solution is referred to as the basic solution. Whenever one or more of the basic variables (decision and/or slack variables) happen to be at one of their bounds, the corresponding basic solution is said to be degenerate.

The simplex algorithm gives extra information in addition to the optimal solution. The algorithm provides marginal values which give information on the variability of the optimal solution to changes in the data. The marginal values are divided into two groups:

- shadow prices which are associated with constraints and their right-hand side, and
- Reduced costs which are associated with the decision variables and their bounds.

These are discussed in the next two sections.

In addition to marginal values, the simplex algorithm can also provide sensitivity range information. These ranges are defined in terms of two of the characteristics of the optimal solution, namely the optimal objective value and the optimal basis.

SHADOW PRICES

"The marginal value of a constraint, referred to as its shadow price, is defined as the rate of change of the objective function from a one unit increase in its right-hand side. Therefore, a positive shadow price indicates that the objective will increase with a unit increase in the right-hand side of the constraint while a negative shadow price indicates that the objective will decrease. For a nonbinding constraint, the shadow price will be zero since its right-hand side is not constraining the optimal solution."

To improve the objective function (that is, decreases for a minimization problem and increases for a maximization problem), it is necessary to weaken a binding constraint. This is intuitive because relaxing is equivalent to enlarging the feasible region. A "≤"constraints weakened by increasing the right-hand side and a "≥"constraints weakened by decreasing the right-hand side. It therefore follows that the signs of the shadow prices for binding inequality constraints of the form "≤"and"≥" are opposite.

When our model includes equality constraints, such a constraint could be incorporated into the LP by converting it into two separate inequality constraints. In this case, at most one of these will have a nonzero price. As discussed above, the nature of the binding constraint can be inferred from the sign of its shadow price. For example, consider a minimization problem with a negative shadow price for an equality constraint. This indicates that the objective will decrease (i.e. improve) with an increase in the right-hand side of the equality constraint. Therefore, it is possible to conclude that it is the "≤" constraint (and not the "≥" constraint) that is binding since it is relaxed by increasing the right-hand side.

REDUCED COSTS

" A decision variable has a marginal value, referred to as its reduced cost, which is defined as the rate of change of the objective function for a one unit increase in the bound of this variable. If a nonbasic variable has a positive reduced cost, the objective function will increase with a one unit increase in the binding bound. The objective function will decrease if a nonbasic variable has a negative reduced cost. The reduced cost of a basic variable is zero since its bounds are nonbinding and therefore do not constrain the optimal solution."

By definition, a nonbasic variable is at one of its bounds. Moving it off the bound when the solution is optimal, is detrimental to the objective function value. A nonbasic variable will improve the objective function value when its binding bound is relaxed. Alternatively, the incentive to include it in the basis can be increased by adjusting its cost coefficient. The next two paragraphs explain how reduced cost information can be used to modify the problem to change the basis.

A nonbasic variable is at either its upper or lower bound. The reduced cost gives the possible improvement in the objective function if its bound is relaxed. Relax means decreasing the bound of a variable at its lower bound or increasing the bound of a variable at its upper bound. In both cases the size of the feasible region is increased.

The objective function value is the summation of the product of each variable by its objective cost coefficient. Therefore, by adjusting the objective cost coefficient of a nonbasic variable it is possible to make it active in the optimal solution. The reduced cost represents the amount by which the cost coefficient of the variable must be lowered. A variable with a positive reduced cost will become active if its cost coefficient is lowered, while the cost coefficient of a variable with a negative reduced cost must be increased.

EXAMPLE OF SENSITIVITY RANGES WITH CONSTANT OBJECTIVE FUNCTION VALUE

Table: presents the shadow prices associated with the constraints in the potato chips example.

Process	Constraint	Optimal time	Upper bound	Shadow price
Slicing	$2x_p + 4x_m \leq 345$	315	345	0.00
Frying	$4x_p + 5x_m \leq 480$	480	480	0.17
packing	$4x_p + 2x_m \leq 330$	330	330	0.33

Table: Shadow prices

Optimal decision variables and shadow prices are not always unique. In this section the range of values of optimal decision variables and optimal shadow prices for the potato chips model is examined. In Aims there are in-built facilities to request such range information.

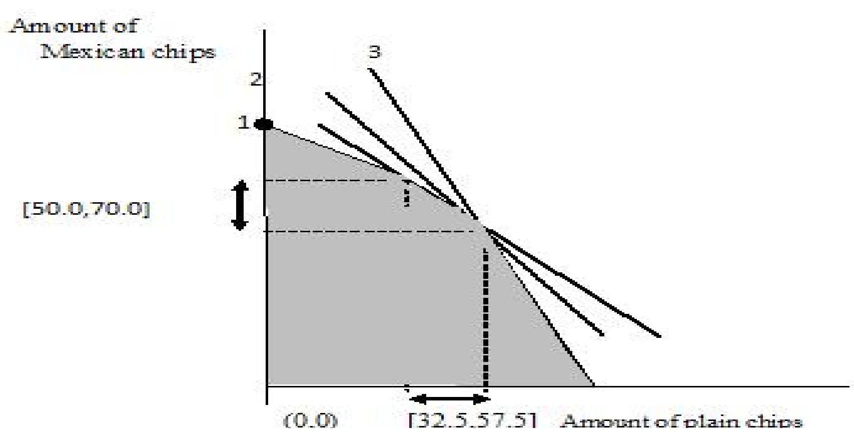

Figure: Decision variable ranges illustrated

This Figure illustrates the sensitivity ranges for decision variables if the three bold lines are interpreted as deferent objective contours. It is clear that for contour (1) there is a range of values for the amount of plain chips ([32.5, 57.5] kg) and, a corresponding range for the amount of Mexican chips ([50.0, 70.0] kg) that can yield the same objective value. Contour (3) also exhibits this behavior but the ranges are deferent. For objective contour (2), there is a unique optimal decision.

The bold lines in Figure were initially interpreted as constraints that intersect at the optimal solution. In this case, the shadow prices are not unique and the situation is referred to as a case of degeneracy. The potato chip problem to date does not have a constraint corresponding to line (2) but a new constraint can easily be added for illustrative purposes only. This constraint limits the objective value to be less than its optimal value. Thus, the contours in Figure can also be interpreted as follows:

i. frying constraint,
ii. new constraint limiting the optimal value, and
iii. Packing constraint.

NATURE AND SOURCES OF DATA:

The data for the thesis were collected from the book *Schedule of rates for civil works* published by *Public work department* (Government of the people's, republic of Bangladesh) and the *Mat Home Limited*, a real estate company. The actual expenditure incurred on material, labour and overheads were obtained from the book *Schedule of rates for civil works* published by *Public work department* and *Mat Home Limited.* This data are used to formulate a linear programming problem and solving it by simplex method, graphical method and using Mat lab that we maximize our benefits with a limited resource.

EFFECTS OF SCALE ON CONSTRUCTION COST

We are now going to estimate cost of the project in Control Estimates manner. In this process we collect data from the book *Schedule of rates for civil works* published by *Public work department* and *Mat Home Limited.* In this project the projected total Budget is Tk. 3962700.
Whose Head of the work as follows in the below:

CIVIL WORKS (per sqf)	Material cost (per sqf) 68%	Labour cost (per sqf) 30%	Transport cost (per sqf) 2%
1200	816	360	24

INTERNAL ELECTRIC WORKS (per sqf)	Material cost (per sqf) 68%	Labour cost (per sqf) 30%	Transport cost (per sqf) 2%
120	82	36	2

SANITARY & WATER SUPPLY (per sqf)	Material cost (per sqf) 68%	Labour cost (per sqf) 30%	Transport cost (per sqf) 2%
70	48	21	1

RESOURCES AVAILABILITY AND PROCEDURE

Constructing building is a large scale project. It's very difficult task to arrange total amount in starting of large project. Sometimes we are failure to arrange it. In this situation we start our work with limited budget. Here we arrange such type of model problem to complete maximum work in our limited budget. a constructing building starting its work with limited budget. after completing design and foundation work we have 2 crore to complete the work. So what will be the maximum floor totally completed in this limited budget. Here we use data from previous chapter. using two variable x_1, x_2. x_1 represents civil works (per floor) and x_2 represents Internal electric work, sanitary & water supply work (per apartment).three types of cost calculate here. Material cost, labour cost and transport cost. we divide our amount as total material cost never exceed Tk 14000000, total labour cost never exceed Tk 5600000 and total transport cost never exceed Tk400000. All costs are collected from Mat home limited on 12 April. civil work means completing ceiling,philar,door,window and color.for civil work the material cost(per floor) is 2496960,labour cost(per floor) is 1101600, transport cost(per floor) is 73440.and for Internal electric work, sanitary & water supply work the material cost(per apartment).is 198900,labour cost(per apartment).is 8721, transport cost(per apartment) 4590.

MATHEMATICAL ANALYSES FOR CONSTRUCTING PROJECT

For our required constructing building, our plot size is ten katha.

We know that,

1 katha = 720 sqf ; [sqf means square feet]

So, 10 katha = 7200 sqf

Useable area: According to the rules of RAZUK, for a residential building we can use 57.5% area of the total area.

In this connection, $7200 of (57.5/100)$ Or, 4140 sqf

We can use maximum 4140sqf area out of 7200 sqf for our constructing project

FAR: FAR means Floor Area Ratio. From RAJUK restriction far must be 4.25. If we want to construct 10 floors, then we can use 3060 sqf per floor out of 4140 sqf. Each floor contains two apartments and each apartment area is (3060/2) = 1530 sqf

Standard maximization problem – a linear programming problem for which the objective function is to be maximized and all the constraints are "less-than-or-equal-to" inequalities.

	Civil work (per floor) BDT	Internal electric work, sanitary & water supply work (per apartment) BDT	Total BDT
Material cost	2496960	198900	2695860
Labour cost	1101600	8721	1188810
Transport cost	73440	4590	78030
Total	3672000	290700	3962700

Table: Mathematical Formulation For Constructing Building **Check**

By the reason of limited money we start our project work with 20000000.this money distribute as follows that total material cost never exeed 14000000,labour cost must be lie on 560000 and for transport cost we can't spent more than 400000.in this limitation we have to find out the maximum completed work.

FORMULATION

To assembly standard simplex method we organize objective function by 'z' and also dispose constraint equation by the following:

Maximize: $3672000x_1 + 290700x_2$
Subject to the constraints,
$2496960\, x_1 + 198900\, x_2 \le 14000000$
$1101600\, x_1 + 87210\, x_2 \le 5600000$
$73440\, x_1 + 4590\, x_2 \le 400000$
$x_1 \le 10$
$-2\, x_1 + x_2 \le -2$
$x_1, x_2 \ge 0$
Introducing slack variables
i.e, Maximize: $3672000x_1 + 290700x_2$
Subject to the constraints,
$2496960\, x_1 + 198900\, x_2 + s_1 = 14000000$
$1101600\, x_1 + 87210\, x_2 + s_2 = 5600000$
$73440\, x_1 + 4590\, x_2 + s_3 = 400000$
$x_1 + s_4 = 10$
$-2\, x_1 + x_2 + s_5 = -2$
$x_1, x_2, s_1, s_2, s_3, s_4, s_5 \ge 0$

where s_1, s_2, s_3, s_4, s_5 are slack variables.

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To solving this linear system using simplex methods we get the values from table

c_0	c_j	3672000	290700	0	0	0	0	0	Constant	Ratio
		x_1	x_2	s_1	s_2	s_3	s_4	s_5		
0	s_1	0	0	1	2.27	0	0	-1055.17	1298035.36	
3672000	x_1	1	0	0	9.08×10^{-7}	0	0	-0.068	4.57	
0	s_3	0	0	0	0.07	1	0	1055.17	35297.98	
0	s_4	0	0	0	-9.08×10^{-7}	0	1	0.068	5.48	
290700	x_2	0	1	0	0	0	0	0.86	7.05	
		0	0	0	3.33	0	0	306		

CALCULATING c_j:

<u>For c_2 :</u>

Now for the range of change Δc_2 of c_2 is given by

$$\max_j\left\{\frac{-(E_j-c_j)}{y_{2j}}, y_{2j}>0\right\} \le \Delta c_2 \le \min_j\left\{\frac{-(E_j-c_j)}{y_{2j}}, y_{2j}<0\right\}$$

$$\Rightarrow \max_j\left\{\frac{-3.33}{9.08\times10^{-7}}\right\} \le \Delta c_2 \le \min_j\left\{\frac{-306}{-0.068}\right\}$$

$$\Rightarrow -3667400.881 \le \Delta c_2 \le 4500$$

Verification of c_2 is

$$\Rightarrow 3672000-3667400.881 \le c_2 \le 3672000+4500$$

$$\Rightarrow 4599.119 \le c_2 \le 3676500$$

$$\boxed{\mathbf{4599.119 \le c_2 \le 3676500}}$$

For c_5 :

Now for the range of change Δc_5 of c_5 is given by

$$\max_j\left\{\frac{-(E_j - c_j)}{y_{5j}}, y_{5j} > 0\right\} \le \Delta c_5 \le \min_j\left\{\frac{-(E_j - c_j)}{y_{5j}}, y_{5j} < 0\right\}$$

$$\Rightarrow \max_j\left\{\frac{-306}{0.86}\right\} \le \Delta c_5$$

$$\Rightarrow -355.81 \le \Delta c_5$$

Verification of c_5 is

$$\Rightarrow 290700 - 355.81 \le c_5$$

$$\Rightarrow 290344.19 \le c_5$$

Which has no upper bound.

$$\boxed{\mathbf{290344.19 \le c_5}}$$

CALCULATING b_i:

The optimal basis is B = ($s_1, x_1, s_3, s_3, s_4, x_2$)

$$\begin{pmatrix} 1 & 2496960 & 0 & 0 & 198900 \\ 0 & 1101600 & 0 & 0 & 87210 \\ 0 & 73440 & 1 & 0 & 4590 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 1 \end{pmatrix}$$

Then y_3, y_4, y_5, y_6, y_7 under s_1, s_2, s_3, s_4, s_5 respectively of the final table 3 in chapter 5 constitute final basis inverse which is given by

$$B^{-1} = \begin{pmatrix} 1 & 2.27 & 0 & 0 & -1055.18 \\ 0 & 9.08\times10^{-7} & 0 & 0 & -0.068 \\ 0 & 0.07 & 1 & 0 & 1055.18 \\ 0 & 9.08\times10^{-7} & 0 & 1 & 0.068 \\ 0 & 0 & 0 & 0 & 0.86 \end{pmatrix}$$

$$x_B = \begin{pmatrix} 1298035.36 \\ 4.57 \\ 35297.98 \\ 5.48 \\ 7.05 \end{pmatrix}$$

We know that,

$$\max_i\left\{\frac{-x_{Bi}}{b_{ik}}, b_{ik} > 0\right\} \le \Delta b_k \le \min_i\left\{\frac{-x_{Bi}}{b_{ik}}, b_{ik} < 0\right\} \quad \text{..................(1)}$$

The individual range of changes of b_1 , b_2 , b_3 , b_4 , b_5 are given by,

For k=1 :

From equation (1), we have

$$\max\{-1298035.36\} \le \Delta b_1$$

$$\Rightarrow -1298035.36 \le \Delta b_1$$

$$\Rightarrow 1400000 - 1298035.36 \le b_1$$

$$\Rightarrow 12701964.64 \le b_1$$

$$\boxed{\mathbf{12701964.64 \le b_1}}$$

For k=2 :

From equation (1), we have

$$\max\left\{\frac{-1298035.36}{2.27}, \frac{-4.57}{9.08\times10^{-7}}, \frac{-35297.98}{0.07}\right\} \le \Delta b_2 \le \min\left\{\frac{5.48}{9.08\times10^{-7}}\right\}$$

$$\Rightarrow \max\{-571821.74, -5033039.65, -504256.86\} \le \Delta b_2 \le \min\{6035242.291\}$$

$$\Rightarrow -5033039.65 \le \Delta b_2 \le 6035242.291$$

$$\Rightarrow 5600000 - 5033039.65 \le b_2 \le 5600000 + 6035242.291$$

$$\Rightarrow 566960.35 \le b_2 \le 11635242.291$$

$$\boxed{\mathbf{566960.35 \le b_2 \le 11635242.291}}$$

For k=3:

From equation (1), we have

$$\max\{-35297.98\} \le \Delta b_3$$

$$\Rightarrow 400000 - -35297.98 \le b_3$$

$$\Rightarrow 364702.02 \le b_3$$

$$\boxed{\mathbf{364702.02 \le b_3}}$$

For k=4:

From equation (1), we have

$$\max\{-5.48\} \le \Delta b_4$$

$$\Rightarrow 10 - 5.48 \le b_4$$

$$\Rightarrow 4.52 \le b_4$$

$$\boxed{\mathbf{4.52 \le b_4}}$$

For k=5:

From equation (1), we have

$$\max\left\{\frac{-35297.98}{1055.17}, \frac{-5.48}{0.068}, \frac{-7.05}{0.86}\right\} \le \Delta b_5 \le \min\left\{\frac{1298035.36}{1055.17}, \frac{4.57}{0.068}\right\}$$

$$\Rightarrow \max\{-33.45, -80.59, -8.2\} \le \Delta b_5 \le \min\{1230.17, 67.21\}$$

$$\Rightarrow -2-8.2 \le b_5 \le -2+67.21$$
$$\Rightarrow -10.2 \le b_5 \le 65.21$$
$$\boxed{\mathbf{-10.2 \le b_5 \le 65.21}}$$

CONCLUSION

The concepts of marginal values and ranges have been explained using the optimal solution of the constructing project. The use of both shadow prices and reduced costs in sensitivity analysis has been demonstrated. Sensitivity ranges have been introduced to provide validity ranges for the optimal objective function value and optimal basis. Although there are some benefit in predicting the effect of changes in data, it has been shown that these indicators do have their limits. Repeated solving of the model provides the best method of sensitivity analysis, and the Aims modeling system has some powerful facilities to support this type of sensitivity analysis. We analyze the sensitivity analysis ,which provide the validity ranges for the optimal objective function value and optimal basis. . Although there is some benefit in predicting the effect of changes in data, it has been shown that these indicators do have their limits. Repeated solving of the model provides the best method of sensitivity analysis, and the Aims modeling system has some powerful facilities to support this type of sensitivity analysis.

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